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A FERMATEAN FUZZY RANKING FUNCTION IN OPTIMIZATION OF INTUITIONISTIC FUZZY TRANSPORTATION PROBLEMS

M.K. Sharma[1], Sadhna[1], A.K. Bhargava[2], Sanjeev Kumar[1], Laxmi Rathour[3], Lakshmi Narayan Mishra[4*], Shikha Pandey[5]

[1]Chaudhary Charan Singh University, Meerut, India
[2]M.M.H. College, Ghaziabad, India
[3]Ward Number 16, Bhagatbandh, Annupur 484 224, Madhya Pradesh, India
[4]Vellore Institute of Technology, Vellore 632 014, Tamil Nadu, India
[5]Jaypee Institute of Information Technology, Ghaziabad, India

Abstract. In the real world, it is not necessary that all the parameters i.e., cost, demand and supply related to the Transportation Problem (TP) are known precisely. One of the recent ways to tackle the impreciseness is the Fermatean fuzzy set (FFS), an extension of Pythagorean fuzzy set (PFS). First, we established a score function for grading FFS in this research paper. The main aim of our research article is to solve the TP in a Fermatean fuzzy environment. In order to optimise the TP using Fermatean fuzzy parameters, we presented an algorithm for three types of Fermatean fuzzy transportation problems (ty-1 FFTP, ty-2 FFTP, and ty-3 FFTP). Numerical examples are also performed to emphasise the suggested method, and the results are compared to the present work. The significance of the work as well as its possibilities in the future has been addressed.

Keywords: Fermatean fuzzy set, Fermatean fuzzy transportation problem, Pythagorean fuzzy set, Ranking function, Transportation problem.

AMS Subject Classification: 03E72, 90C08.

Corresponding author: Lakshmi Narayan, Mishra, Vellore Institute of Technology, Vellore 632 014, Tamil Nadu, India, e-mail: *lakshminarayanmishra04@gmail.com*

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1 Introduction

In the dynamics our highly competitive market, searching the most efficient way to produce and transport to customers has become a difficult issue. In order to supply goods to the customers, the management of the transport network encounters numerous challenges that includes handling of technological changes, enhancing productivity, globalization, changing customer's expectations, market dynamics and safety issues. But in the present scenario, to deal these problems become exhausting. Transport system consists of a significant framework to meet these types of difficulties and assures the right time delivery of different form of articles.

The Transportation Problem (TP) was first proposed by Hitchcock (1941) to keep the demand flowing from origin to their terminal. TP is a Linear Programming Problem (LPP) that arises from a framework with a fixed numerals of nodes and arcs. In TP either the total profit is maximized or the total transport cost is minimized. The stepping stone method to solve the TP was introduced by Charnes et al (1953). Thereafter, Dantzig (1963) initiated the primal simplex transportation scheme. There are several approaches to find the initial basic feasible solution of a TP and various evolutionary algorithms are also available for dealing with TPs in the present literature.

In real life problems the entire factor's related to the TP are generally imprecise or vague due to the uncertainty and lack of complete information. To deal with uncertainty in decision making, Zadeh (1965), introduced the notion of fuzzy logic. Thereafter, Bellman & Zadeh (1970), pioneered the theory of uncertain decision-making tasks. Zimmermann (1978) demonstrated that fuzzy LP solutions are always optimal and effective.

Thereafter, Chanas et al. (1984) proposed a fuzzy LP model to address the TP with crisp cost, fuzzy supply, and demand. Chanas & Kuchata (1996), explored the form of the TP with imprecise parameters. Following that, various experts have formulated the TP using either a single objective TP or a multi-objective (MOTP) model, while taking into account various fuzzy contexts. Tada & Ishii (1996), Kaur & Kumar (2011), Liu et al. (2014), Kundu et al. (2014), Singh & Yadav (2016), Gupta & Anupum (2017), Arora (2018), Bharati & Singh (2018), Kumar (2018), and Hashmi et al. (2019) have worked on single objective TP.

In the context of the MOTP, Li & Lai (2000) introduced an imprecise compromise programming technique to the MOTP. In addition to imprecise factors, Ahmad & Adhami (2019) considered nonlinearity in MOTPs, yielding in nonlinear membership for every objective function in a neutrosophic system. A multi-objective transportation-p-facility localization issue was presented by Das & Roy (2019). To establish a real transport network, Das et al. (2020) incorporated a two-fold (type-2 intuitionistic) ambiguity. Ghosh et al. (2021) developed a MOSTP with fixed in the intuitionistic fuzzy context. Then, in TP, Ghosh & Roy (2021) examined at an additional cost that considers as type-II fixed-charge and truck load limits. Midya et al. (2021) developed a MOSTP with fixed charge and various stages in an intuitionistic fuzzy setting with a green supply network.

In 1986, Atanassov (1986) initiated the notion of Intuitionistic Fuzzy Set (IFS) and incorporated both membership ϕ and non-membership ψ functions with hesitation margin π in the following manner $\phi + \psi \leq 1$ and $\phi + \psi + \pi = 1$. However, the total of the membership and non-membership functions can exceed 1 in a lot of circumstances. Thus, Yager (2013) recently initiated the notion Pythagorean Fuzzy Set (PFS), a development of IFS.

Some real-life applications under Pythagorean fuzzy environment proposed by various researchers are given as follows: Zhang & Xu (2014) developed some innovative PFS operating laws and presented an enhanced rank preference approach in 2014. In addition, Mohd & Abdullah (2017) introduced the Pythagorean fuzzy analytic hierarchy method to establish the assessment criteria weight. Then, Kumar et al. (2019) proposed optimization models for TP under Pythagorean fuzzy environment. Jeyalakshmi et al. (2021) introduced the Monalisha method to solve TP with PFS parameters. Prabha (2021), introduced a geometric mean method for solving TP in Pythagorean fuzzy settings.

However, in real-life applications, there may be a situation, for instance when 0.9 and 0.6 are in support and against support for membership in the FS. As, it violates both the IFS and PFS constraint conditions. Thus, Senapati & Yager (2020), Senapati & Yager (2019) introduced the notion of Fermatean Fuzzy Set (FFS) to tackle these types of situations in decision making and also compared FFSs with PFSs and IFSs. Then, Sahoo (2021) initially solved TP in Fermatean fuzzy environment and proposed an algorithm in which Fermatean fuzzy transportation problem (FFTP) is converted into conventional TP which is then solved by using excel solver in order to find out best solution. Thereafter, Sahoo (2021) proposed three score functions for the ranking of FFS and also applied TOPOSIS method to solve the multi-criterion decision-making problem in Fermatean fuzzy context. Then, Kumar et al. (2019) proposed a family of third-order derivative-free iterative methods for multiple zeros. Also, for approximating the repeated roots of non-linear equations, Kumar et al. (2020) proposed a derivative-free numerical method with optimal fourth order convergence.

We primarily proposed a score function for the defuzzification of FFS in this research work. Furthermore, a methodology is developed to tackle the TP of three different types in Fermatean fuzzy settings. In proposed method, all three types of Fermatean Fuzzy Transportation Problem

(FFTP) are first converted into conventional TP with the help of new score function. Thereafter, the conventional TP been solved by using proposed algorithm to get the optimum result. In the end, in order to demonstrate the proposed method, we solved numerical examples, for each type of FFTP and also comparison results with the existing literature shows that the proposed techniques provide better results than the existing ones.

The work done in this research article is categorized into seven sections and organized as follows: first section of the paper comprises a brief introduction. In section two, some elementary definitions of FFS theory and their operations are presented. In section three, we proposed a score function. The mathematical formulation for three types of FFTP and a proposed algorithm to unravel the TPs in Fermatean fuzzy environment tare presented in section four. The results of different types of FFTP are discussed in section five. Finally, the inferences for the work done based on the results are drawn in section six. All the required abbreviations are given in Table 1.

Table 1: List of abbreviations

Notations	Full Form
FFS	Fermatean Fuzzy Set
FFTP	Fermatean Fuzzy Transportation Problem
FTP	Fuzzy Transportation Problem
IFS	Intuitionistic Fuzzy Set
PFS	Pythagorean Fuzzy Set
ty-1FFTP	type 1 Fermatean Fuzzy Transportation Problem
ty-2 FFTP	type 2 Fermatean Fuzzy Transportation Problem
ty-3 FFTP	type 3 Fermatean Fuzzy Transportation Problem

2 Preliminaries

Several basic definitions regarding FFSs Senapati & Yager (2020), Senapati & Yager (2019), Sahoo (2021) are provided in this section in a modified form.

2.1 Fermatean fuzzy set (FFS)

A Fermatean Fuzzy Set (FFS) on a universal set X is a denoted as:

$$\tilde{F} = \{\langle x, \phi_{\tilde{F}}(x), \psi_{\tilde{F}}(x) : x \in X\rangle\}$$

$s.t \phi_{\tilde{F}}(x) : X \to [0,1]$ and $\psi_{\tilde{F}}(x) : X \to [0,1]$ with $0 \leq (\phi_{\tilde{F}}(x))^3 + (\psi_{\tilde{F}}(x))^3 \leq 1, \forall x \in X$. Also $\phi_{\tilde{F}}(x)$ and $\psi_{\tilde{F}}(x)$ represents the degree of membership and non-membership of $x \in X$ in $\tilde{F}$.

The degree of indeterminacy for any FFS $\tilde{F}$ for $x \in X$ is represented by

$$\xi_F(x) = \sqrt[3]{1 - (\phi_{\tilde{F}}(x))^3 - (\phi_{\tilde{F}}(x))^3}.$$

The set $\tilde{F} = \{\langle x, \phi_{\tilde{F}}(x), \psi_{\tilde{F}}(x) : x \in X\rangle\}$ is denoted as $\tilde{F} = \langle \phi_{\tilde{F}}, \psi_{\tilde{F}}\rangle$ due to explicitness.

2.2 Elementary Operations on FFS

Let us consider three FFS $\tilde{F} = \langle \phi_{\tilde{F}}, \psi_{\tilde{F}}\rangle$, $\tilde{F_1} = \langle \phi_{\tilde{F_1}}, \psi_{\tilde{F_1}}\rangle$ and $\tilde{F_2} = \langle \phi_{\tilde{F_2}}, \psi_{\tilde{F_2}}\rangle$ on universal set X and $\lambda > 0$.The elementary operations on the FFS are defined as follows:

1. Addition:$\tilde{F_1} \oplus \tilde{F_2} = \langle \sqrt[3]{(\phi_{\tilde{F_1}})^3 + (\phi_{\tilde{F_2}})^3 - (\phi_{\tilde{F_1}})^3(\phi_{\tilde{F_2}})^3}, \psi_{\tilde{F_1}}\psi_{\tilde{F_2}}\rangle$

2. Multiplication:$\tilde{F_1} \otimes \tilde{F_2} = \langle \phi_{\tilde{F_1}} \phi_{\tilde{F_2}}, \sqrt[3]{(\psi_{\tilde{F_1}})^3 + (\psi_{\tilde{F_2}})^3 - (\psi_{\tilde{F_1}})^3 (\psi_{\tilde{F_2}})^3} \rangle$

3. Scalar Multiplication:$\lambda \odot \tilde{F} = \langle \sqrt[3]{1 - (1 - \phi_{\tilde{F}}^3)^\lambda}, (\psi_{\tilde{F}})^\lambda \rangle$

4. Exponent:$\tilde{F}^\lambda = \langle (\phi_{\tilde{F}})^\lambda, \sqrt[3]{1 - (1 - \psi_{\tilde{F}}^3)^\lambda} \rangle$

5. Union: $\tilde{F_1} \cup \tilde{F_2} = \langle max(\phi_{\tilde{F_1}}, \phi_{\tilde{F_2}}), min(\psi_{\tilde{F_1}}, \psi_{\tilde{F_2}}) \rangle$

6. Intersection:$\tilde{F_1} \cap \tilde{F_2} = \langle min(\phi_{\tilde{F_1}}, \phi_{\tilde{F_2}}), max(\psi_{\tilde{F_1}}, \psi_{\tilde{F_2}}) \rangle$

7. Complement: $\tilde{F}^C = \langle \psi_{\tilde{F}}, \phi_{\tilde{F}} \rangle$

Example 1. *Let* $\tilde{F} = \langle 0.8, 0.3 \rangle, \tilde{F_1} = \langle 0.4, 0.7 \rangle, \tilde{F_2} = \langle 0.5, 0.4 \rangle$ *be three FFS and* $\lambda = 2$ *be a scalar, then: -*

$$\begin{aligned}
(i)\tilde{F_1} \oplus \tilde{F_2} &= \langle 0.4, 0.7 \rangle \oplus \langle 0.5, 0.4 \rangle \\
&= \langle 0.0026, 0.28 \rangle \text{ (Using definition 2.2)} \\
(ii)\tilde{F_1} \otimes \tilde{F_2} &= \langle 0.4, 0.7 \rangle \otimes \langle 0.5, 0.4 \rangle \\
&= \langle 0.20, 0.5723 \rangle \text{ (Using definition 2.2)} \\
(iii)\lambda \odot \tilde{F} &= 2 \odot \langle 0.8, 0.3 \rangle \\
&= \langle 0.07938, 0.09 \rangle \text{ (Using definition 2.2)} \\
(iv)\tilde{F}^\lambda &= \langle 0.8, 0.3 \rangle^2 \\
&= \langle 0.64, 0.4481 \rangle \text{ (Using definition 2.2)} \\
(v)\tilde{F_1} \cup \tilde{F_2} &= \langle 0.4, 0.7 \rangle \cup \langle 0.5, 0.4 \rangle \\
&= \langle 0.5, 0.4 \rangle \text{ (Using definition 2.2)} \\
(vi)\tilde{F_1} \cap \tilde{F_2} &= \langle 0.4, 0.7 \rangle \cap \langle 0.5, 0.4 \rangle \\
&= \langle 0.4, 0.7 \rangle \text{ (Using definition 2.2)} \\
(vii)\tilde{F}^c &= \langle 0.8, 0.3 \rangle^c \\
&= \langle 0.3, 0.8 \rangle \text{ (Using definition 2.2)}
\end{aligned}$$

2.3 Score Function of FFS

Let us consider an FFS $\tilde{F} = \langle \phi_{\tilde{F}}, \psi_{\tilde{F}} \rangle$. Then the score function for $\tilde{F}$ is symbolized as $S_F(\tilde{F})$ and described in the following manner:

$$S_F(\tilde{F}) = (\phi_{\tilde{F}}^3 - \psi_{\tilde{F}}^3).$$

2.4 Accuracy Function of FFS

Let us consider an FFS $\tilde{F} = \langle \phi_{\tilde{F}}, \psi_{\tilde{F}} \rangle$. Then the score function for $\tilde{F}$ is symbolized as $A_F(\tilde{F})$ and described in the following manner:

$$A_F(\tilde{F}) = (\phi_{\tilde{F}}^3 + \psi_{\tilde{F}}^3).$$

3 Proposed Fermatean Fuzzy Score Function

In this section, we developed a ranking function for the ordering of the FFS in decision making problems.

3.1 Fermatean fuzzy Score function

Let us consider an FFS $\tilde{F} = \langle \phi_{\tilde{F}}, \psi_{\tilde{F}} \rangle$. Then the score function for $\tilde{F}$ is symbolized as follows:

$$S_F^*(\tilde{F}) = \frac{1}{2}(1 + \phi_{\tilde{F}}^2 - \psi_{\tilde{F}}^2) \cdot (min(\phi_{\tilde{F}}, \psi_{\tilde{F}})).$$

Property 1. *Consider an FFS $\tilde{F} = \langle \phi_{\tilde{F}}, \psi_{\tilde{F}} \rangle$, then $S_F^*(\tilde{F}) \in [0,1]$.*

Proof. By the definition of an ortho pair, $\phi_{\tilde{F}}, \psi_{\tilde{F}} \in [0,1]$. Then, $min(\phi_{\tilde{F}}, \psi_{\tilde{F}}) \in [0,1]$. Also, $\phi_{\tilde{F}}^2 \geq 0, \psi_{\tilde{F}}^2 \geq 0, \phi_{\tilde{F}}^2 \leq 1$ and $\psi_{\tilde{F}}^2 \leq 1$

$$\begin{aligned}
&\implies 1 - \psi_{\tilde{F}}^2 \geq 0 \\
&\implies 1 + \phi_{\tilde{F}}^2 - \psi_{\tilde{F}}^2 \geq 0 \\
&\therefore \frac{1}{2}(1 + \phi_{\tilde{F}}^2 - \psi_{\tilde{F}}^2) \cdot (min(\phi_{\tilde{F}}, \psi_{\tilde{F}})) \geq 0 \\
&\text{Again, } \phi_{\tilde{F}}^2 - \psi_{\tilde{F}}^2 \leq 1 \\
&\implies 1 + \phi_{\tilde{F}}^2 - \psi_{\tilde{F}}^2 \leq 2 \qquad (\because \phi_{\tilde{F}}^2 \geq 0) \\
&\implies \frac{1}{2}(1 + \phi_{\tilde{F}}^2 - \psi_{\tilde{F}}^2) \cdot (min(\phi_{\tilde{F}}, \psi_{\tilde{F}})) \leq 1 \qquad (\because min(\phi_{\tilde{F}}, \psi_{\tilde{F}}) \leq 1) \\
&\text{Hence, } S_F^*(\tilde{F}) \in [0,1].
\end{aligned}$$

□

3.2 Ranking of Fermatean fuzzy sets

Let us consider two FFSs $\tilde{F}_1 = \langle \phi_{\tilde{F}_1}, \psi_{\tilde{F}_1} \rangle$ and $\tilde{F}_2 = \langle \phi_{\tilde{F}_2}, \psi_{\tilde{F}_2} \rangle$, the ranking laws of $\tilde{F}_1$ and $\tilde{F}_2$ are described in the following way:

Case-1: when $S_F^*(\tilde{F}_1) \geq S_F^*(\tilde{F}_2)$ with $A_F^*(\tilde{F}_1) > A_F^*(\tilde{F}_2)$ iff $\tilde{F}_1 \succ \tilde{F}_2$.

Case-2: when $S_F^*(\tilde{F}_1) \leq S_F^*(\tilde{F}_2)$ with $A_F^*(\tilde{F}_1) < A_F^*(\tilde{F}_2)$ iff $\tilde{F}_1 \prec \tilde{F}_2$.

Case-3: when $S_F^*(\tilde{F}_1) = S_F^*(\tilde{F}_2)$ with $A_F^*(\tilde{F}_1) < A_F^*(\tilde{F}_2)$ iff $\tilde{F}_1 = \tilde{F}_2$.

Example 2. *Let $\tilde{F}_1 = \langle 0.4, 0.7 \rangle$ and $\tilde{F}_2 = \langle 0.5, 0.4 \rangle$ be two FFSs, then we have the following:*
We have $S_F^*(\tilde{F}_1) = \frac{1}{2}(1 + 0.4^2 - 0.7^2) \cdot (min(0.4, 0.7)) = 0.134$
and $S_F^*(\tilde{F}_2) = \frac{1}{2}(1 + 0.5^2 - 0.4^2) \cdot (min(0.5, 0.4)) = 0.218$ (By using score function $S_F^*(\tilde{F}) = \frac{1}{2}(1 + \phi_{\tilde{F}}^2 - \psi_{\tilde{F}}^2) \cdot (min(\phi_{\tilde{F}}, \psi_{\tilde{F}}))$).
Hence $S_F^*(\tilde{F}_1) < S_F^*(\tilde{F}_2) \implies \tilde{F}_1 < \tilde{F}_2$.

4 Proposed Fermatean Fuzzy TP model and its algorithm

In this section, first we develop mathematical model for three types TP in Fermatean fuzzy settings and then an algorithm is also proposed in order to tackle the three types of FFTP.

4.1 Mathematical Formulation of the Model

Consider a TP containing m origin points and n terminal points, where in i^{th} origin point convey $a_i > 0$ units and j^{th} terminal point needs $b_j > 0$ units. There is a unit shipping cost c_{ij} for transport linked with each connection (i,j) from origin point i to terminal j. The challenge is to find a cost-effective solution to convey the accessible quantity to meet requirement while minimising total transport price.

The number of units to be transferred from origin i to terminal j is denoted by x_{ij}. The conventional TP is represented mathematically as:

$$\min z_0 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$
$$\sum_{j=1}^{n} x_{ij} \leq a_i,\ i=1,...,m$$
$$\sum_{i=1}^{m} x_{ij} \geq b_j,\ j=1,...,n$$
$$x_{ij} \geq 0,\ \forall i,j \qquad (1)$$

When any of c_{ij}, a_i and b_j is FFS or even if all of them are is taken as Fermatean fuzzy parameters, the conventional TP in (1) is FFTP.The FFS are of the form $\langle \phi, \psi \rangle$, where $0 \leq (\phi)^3 + (\psi)^3 \leq 1$. Let $\langle \phi_{\tilde{a}_i}, \psi_{\tilde{a}_i} \rangle, \langle \phi_{\tilde{b}_j}, \psi_{\tilde{b}_j} \rangle$ and $\langle \phi_{\tilde{c}_{ij}}, \psi_{\tilde{c}_{ij}} \rangle$ be the Fermatean fuzzy availability, requirement and unit transport cost of the article.

Additionally, when the cost parameter c_{ij} in TP (1) are replaced into Fermatean fuzzy parameters, i.e., $\tilde{c}_{ij}^F = \langle \phi_{\tilde{c}_{ij}}, \psi_{\tilde{c}_{ij}} \rangle$, then the newly modelled TP is type-1 Fermatean fuzzy Transportation Problem (ty-1 FFTP). The ty-1 FFTP can be formulated mathematically as follows:

$$\text{Min } \langle \phi_{\tilde{z_0}}, \psi_{\tilde{z_0}} \rangle = \sum_{i=1}^{m}\sum_{j=1}^{n} \langle \phi_{\tilde{c}_{ij}}, \psi_{\tilde{c}_{ij}} \rangle \bigodot x_{ij}$$
$$\text{s.t.} \sum_{j=1}^{n} x_{ij} \leq a_i,\ i=1,..,m$$
$$\sum_{i=1}^{m} x_{ij} \geq b_j,\ j=1,..,n$$

where $x_{ij} \geq 0$ and $0 \leq (\phi_{\tilde{c}_{ij}})^3 + (\psi_{\tilde{c}_{ij}})^3 \leq 1, \quad (2)$

Here, if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, then ty-1 FFTP (2) is balanced FFTP.

Now, when the parameters a_i and b_j of TP (1) are considered as Fermatean fuzzy parameters, i.e., $\tilde{a}_i^F = \langle \phi_{\tilde{a}_i}, \psi_{\tilde{a}_i} \rangle$ and $\tilde{b}_i^F = \langle \phi_{\tilde{b}_j}, \psi_{\tilde{b}_j} \rangle$ respectively, then the newly modelled transportation problem (TP) is as type-2 Fermatean fuzzy transportation problem (ty-2 FFTP). The ty-2 FFTP can be formulated mathematically as follows:

$$\text{Min } \langle \phi_{\tilde{z_0}}, \psi_{\tilde{z_0}} \rangle = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \bigodot x_{ij}$$
$$\text{s.t.} \sum_{j=1}^{n} x_{ij} \leq \langle \phi_{\tilde{a}_i}, \psi_{\tilde{a}_i} \rangle,\ i=1,..,m$$
$$\sum_{i=1}^{m} x_{ij} \geq \langle \phi_{\tilde{b}_j}, \psi_{\tilde{b}_j} \rangle,\ j=1,..,n$$

where $\quad 0 \leq (\phi_{\tilde{a}_i})^3 + (\psi_{\tilde{a}_i})^3 \leq 1,\ 0 \leq (\phi_{\tilde{b}_j})^3 + (\psi_{\tilde{b}_j})^3 \leq 1,$
and $\qquad x_{ij} \geq 0 \quad (3)$
here, when $\sum_{i=1}^{m} \bigoplus \langle \phi_{\tilde{a}_i}, \psi_{\tilde{a}_i} \rangle = \sum_{j=1}^{n} \bigoplus \langle \phi_{\tilde{b}_j}, \psi_{\tilde{b}_j} \rangle$ then ty-2 FFTP (3) is called to be balanced FFTP with $\sum \bigoplus$ denoting the summation of FFS.

In the end, when all the parameters i.e., costs, demand and supply associated with TP(1) are considered to be Fermatean fuzzy parameters, then the problem is type-3 Fermatean fuzzy transportation problem (ty-3 FFTP).

The ty-3 FFTP can be formulated mathematically as follows:

$$\text{Min } \langle \phi_{\tilde{z}_0}, \psi_{\tilde{z}_0} \rangle = \sum_{i=1}^{m} \sum_{j=1}^{n} \langle \phi_{\tilde{c}_{ij}}, \psi_{\tilde{c}_{ij}} \rangle \bigodot x_{ij}$$

$$\text{s.t.} \sum_{j=1}^{n} x_{ij} \leq \langle \phi_{\tilde{a}_i}, \psi_{\tilde{a}_i} \rangle, \ i = 1, .., m$$

$$\sum_{i=1}^{m} x_{ij} \geq \langle \phi_{\tilde{b}_j}, \psi_{\tilde{b}_j} \rangle, \ j = 1, .., n$$

where $0 \leq (\phi_{\tilde{a}_i})^3 + (\psi_{\tilde{a}_i})^3 \leq 1, \ i = 1, ..., m,$
$0 \leq (\phi_{\tilde{b}_j})^3 + (\psi_{\tilde{b}_j})^3 \leq 1, \ j = 1, .., n,$
$x_{ij} \geq 0$ and $0 \leq (\phi_{\tilde{c}_{ij}})^3 + (\psi_{\tilde{c}_{ij}})^3 \leq 1, \quad (4)$
here, if $\sum_{i=1}^{m} \bigoplus \langle \phi_{\tilde{a}_i}, \psi_{\tilde{a}_i} \rangle = \sum_{j=1}^{n} \bigoplus \langle \phi_{\tilde{b}_j}, \psi_{\tilde{b}_j} \rangle$, then ty-3 FFTP (4) is balanced FFTP with $\sum \bigoplus$ denoting the summation of FFS.

4.2 Algorithm for the Fermatean Fuzzy Score Function Based Model

We proposed an algorithm in order to tackle TP under Fermatean fuzzy environment in this section 4.2. The flow chart of our developed algorithm is given in Figure 1. The following are the steps of the developed algorithm:

Step I: Identify type of Fermatean fuzzy transportation problem (FFTP).

- (a): when the FFTP is of type-1, find the score value for every Fermatean fuzzy cost using the stated score function S^*.Formulate the conventional TP by replacing all the Fermatean fuzzy cost by their respective score values.
- (b): when the FFTP is of type-2, find the score value for every Fermatean fuzzy availability and demand using the stated score function S^*.Formulate the conventional TP by replacing all of them by their respective score values.
- (c): when the FFTP is ty-3 FFTP,find the score value for every Fermatean fuzzy cost, availability and demand using the stated score function S^*.Formulate the conventional TP by replacing all the parameters by their respective score values.

Step II: Examine whether the TP is balance or not and we proceed as follows:
when the TP is balance i.e., $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ or Demand = Supply, then move to step III otherwise;
If $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$, then convert the unbalanced TP into balanced TP by adding the dummy for demand or supply.
Then, proceed with this balanced TP.

Step III: To discover the initial basic feasible solution of the conventional TP, the VAM approach is applied.

Step IV: The Modified Distribution Method (MODI) is used to test the conventional TP's optimality.

Step V: In order to get optimal transportation cost, put all the x_{ij} in the objective function of conventional TP.

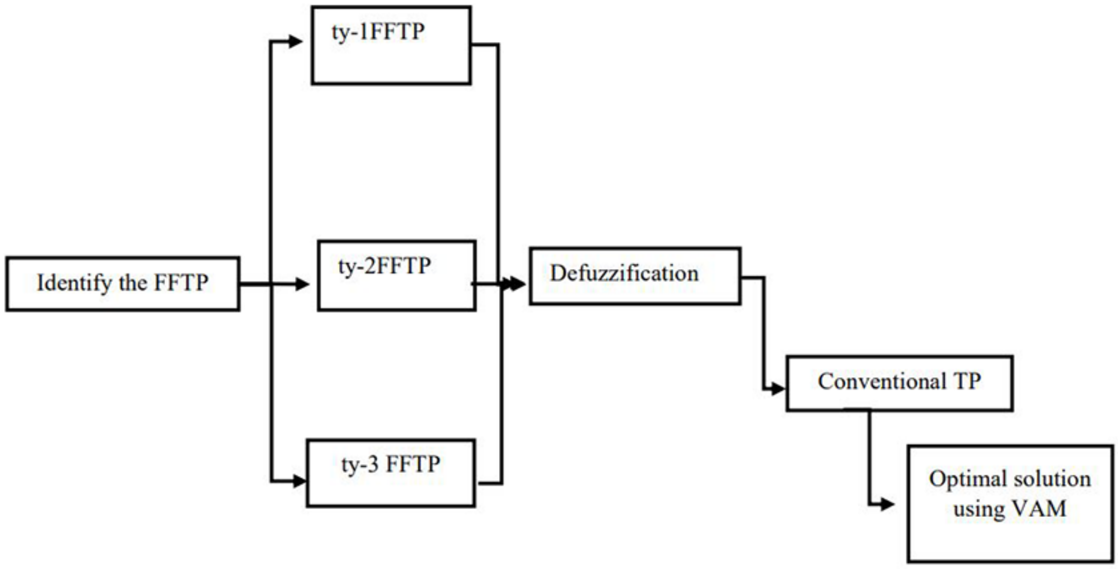

Figure 1: Flow chart for our developed algorithm

5 Numerical Computations

In this section, some examples of all three types of FFTP are carried out from Kumar et al. (2019), to demonstrate the capability of our proposed score function and algorithm.

5.1 Example (ty-1 FFTP)

Consider a TP with input taken from Kumar et al. (2019) of ty-1 PFTP, given in table 2. In order to solve the given example of ty-1 FFTP, use the proposed algorithm.

Table 2: Input for ty-1 FFTP

	T_1	T_2	T_3	T_4	Supply
O_1	$\langle 0.4, 0.7\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.8, 0.3\rangle$	$\langle 0.6, 0.3\rangle$	26
O_2	$\langle 0.4, 0.2\rangle$	$\langle 0.7, 0.3\rangle$	$\langle 0.4, 0.8\rangle$	$\langle 0.7, 0.3\rangle$	24
O_3	$\langle 0.7, 0.1\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.6, 0.4\rangle$	$\langle 0.9, 0.1\rangle$	30
Demand	17	23	28	12	

Now, find the score value of each Fermatean fuzzy cost with the help of proposed score function S^*, formulate a conventional TP by replacing all the parameters by their respective score values. The input data in crisp form of conventional TP is given in table 3.

Table 3: Conventional TP

	T_1	T_2	T_3	T_4	Supply
O_1	0.134	0.218	0.2325	0.1905	26
O_2	0.112	0.21	0.104	0.21	24
O_3	0.074	0.0815	0.24	0.09	30
Demand	17	23	28	12	

The optimal solution of this ty-1 FFTP by using the proposed algorithm is as follows: $x_{11} = 17, x_{13} = 4, x_{14} = 5, x_{23} = 24, x_{32} = 23$ and $x_{34} = 7$.

Therefore, the minimum cost of the ty-1 FFTP is 9.161.

5.2 Example (ty-2 FFTP)

Consider a TP with input from Kumar et al. (2019) of ty-2 PFTP, given in table 4 such that costs are definite, while demand and supply are Fermatean fuzzy factors. In order to solve the given example of ty-2 FFTP, use the proposed algorithm.

Table 4: Input for ty-2 FFTP

	T_1	T_2	T_3	T_4	Supply
O_1	0.0335	0.0545	0.0775	0.0635	$\langle 0.7, 0.1\rangle$
O_2	0.056	0.07	0.026	0.07	$\langle 0.8, 0.1\rangle$
O_3	0.074	0.0815	0.06	0.09	$\langle 0.9, 0.1\rangle$
Demand	$\langle 0.4, 0.7\rangle$	$\langle 0.7, 0.3\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.60832, 0.4\rangle$	

Now, find the score value of each Fermatean fuzzy demand and supply units with the help of proposed score function S^*, formulate a conventional TP by replacing all of them by their respective score values. The input data in crisp form of conventional TP is given in table 5.

Table 5: Conventional TP

	T_1	T_2	T_3	T_4	Supply
O_1	0.0335	0.0545	0.0775	0.0635	0.074
O_2	0.056	0.07	0.026	0.07	0.0815
O_3	0.074	0.0815	0.06	0.09	0.09
Demand	0.134	0.21	0.0815	0.242	

The optimal solution of this ty-2 FFTP by using the proposed algorithm is as follows: $x_{11} = 0.074, x_{23} = 0.0815, x_{31} = 0.06, x_{32} = 0.03, x_{42} = 0.18$ and $x_{44} = 0.242$. Therefore, the minimum cost of the ty-2 FFTP is 0.011483.

5.3 Example (ty-3 FFTP)

Consider a TP with input taken from Kumar et al. (2019) of ty-3 PFTP such that all factors i.e., costs, demand and supply associated with TP are Fermatean fuzzy parameters. In order to solve the given example of ty-3 FFTP, use the proposed algorithm.

Table 6: Input for ty-3 FFTP

	T_1	T_2	T_3	T_4	Supply
O_1	$\langle 0.1, 0.9\rangle$	$\langle 0.2, 0.8\rangle$	$\langle 0.1, 0.8\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.7, 0.1\rangle$
O_2	$\langle 0.01, 0.99\rangle$	$\langle 0.3, 0.9\rangle$	$\langle 0.3, 0.8\rangle$	$\langle 0.1, 0.7\rangle$	$\langle 0.8, 0.1\rangle$
O_3	$\langle 0.1, 0.8\rangle$	$\langle 0.1, 0.8\rangle$	$\langle 0.4, 0.9\rangle$	$\langle 0.2, 0.9\rangle$	$\langle 0.9, 0.1\rangle$
Demand	$\langle 0.4, 0.7\rangle$	$\langle 0.7, 0.3\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.60832, 0.4\rangle$	

Now, using the proposed score function S^*, determine the score value of each Fermatean fuzzy cost, demand, and supply, formulate a conventional TP by replacing all of the parameters by their respective score values. The input data in crisp form of conventional TP is given in table 7.

Table 7: Conventional TP

	T_1	T_2	T_3	T_4	Supply
O_1	0.01	0.04	0.0185	0.01	0.074
O_2	0.0001	0.042	0.0675	0.26	0.0815
O_3	0.0185	0.104	0.07	0.023	0.09
Demand	0.134	0.21	0.0815	0.242	

The optimal solution of this ty-3 FFTP by using the proposed algorithm is as follows: x_{14}=0.074, x_{21}=0.0815, x_{34}=0.09, x_{41}=0.0525, x_{42}=0.21, x_{43}=0.0815 and x_{44}=0.078.
Therefore, the minimum cost of the ty-3 FFTP is 0.002818.

Table 8: Comparison results of our proposed methodology with method proposed by Kumar et al. (2019)

Method	Type	Score function	Optimal solution	Minimum cost
Proposed Method	ty-1 FFTP	$\frac{1}{2}(1+\phi^2_{\tilde{F}}-\psi^2_{\tilde{F}})\cdot(min(\phi_{\tilde{F}},\psi_{\tilde{F}}))$	$x_{11}=17$,$x_{13}=4$, $x_{14}=5$, $x_{23}=24$, $x_{32}=23$, $x_{34}=7$	9.161
	ty-2 FFTP	$\frac{1}{2}(1+\phi^2_{\tilde{F}}-\psi^2_{\tilde{F}})\cdot(min(\phi_{\tilde{F}},\psi_{\tilde{F}}))$	$x_{11}=0.074$,$x_{23}=0.0815$, x_{31} =0.06, $x_{32}=0.03$, $x_{42}=0.18$, $x_{44}=0.242$	0.011483
	ty-3 FFTP	$\frac{1}{2}(1+\phi^2_{\tilde{F}}-\psi^2_{\tilde{F}})\cdot(min(\phi_{\tilde{F}},\psi_{\tilde{F}}))$	$x_{14}=0.074$,$x_{21}=0.0815$, x_{34} =0.09, $x_{41}=0.0525$, $x_{42}=0.21$, $x_{43}=0.0815$, $x_{44}=0.078$	0.002818
Method proposed by Kumar et al. (2019)	ty-1 PFTP	$\frac{1}{2}(1+\phi^2_{\tilde{F}}-\psi^2_{\tilde{F}})$	$x_{11}=17$, $x_{12}=9$, $x_{23}=24$, $x_{32}=14$, $x_{33}=4$, $x_{34}=12$	41.45
	ty-2 FFTP	$\frac{1}{2}(1+\phi^2_{\tilde{F}}-\psi^2_{\tilde{F}})$	$x_{11}=0.3350$, $x_{12}=0.405$, x_{23} =0.8150, $x_{32}=0.295$, $x_{34}=0.6050$	0.132978
	ty-3 FFTP	$\frac{1}{2}(1+\phi^2_{\tilde{F}}-\psi^2_{\tilde{F}})$	$x_{11}=0.3350$, $x_{14}=0.52$, x_{21}=0.3350, $x_{22}=0.48$, $x_{33}=0.815$, $x_{34}=0.085$	0.31895

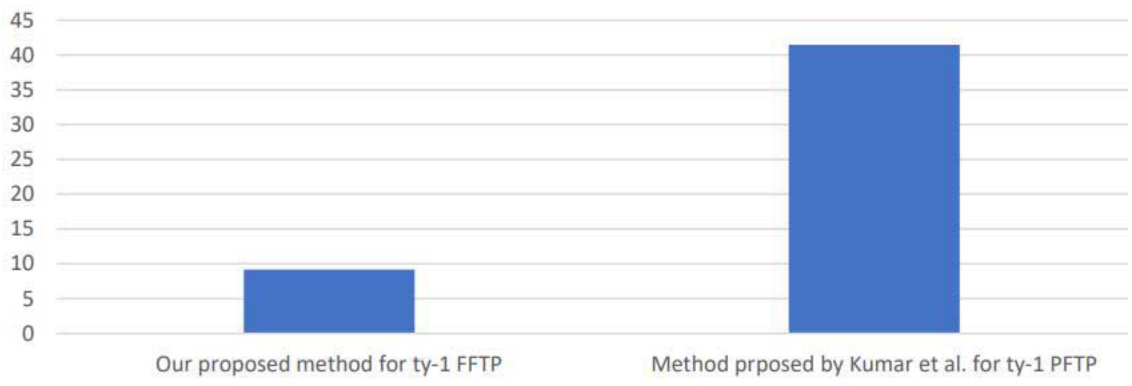

Figure 2: Comparison graph for our proposed method with existing method for ty-1.

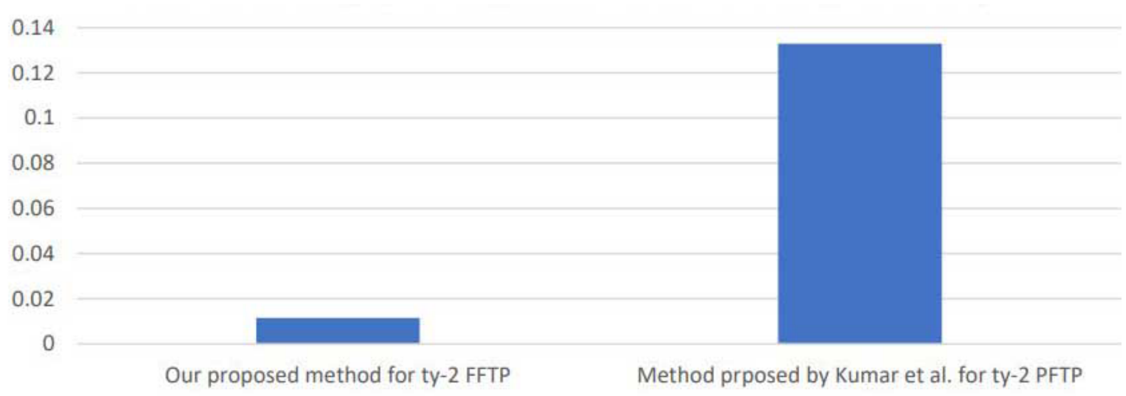

Figure 3: Comparison graph for our proposed method with existing method for ty-2.

Figure 4: Comparison graph for our proposed method with existing method for ty-3.

Table 8 shows that utilising the suggested technique, we were able to acquire the lowest transportation cost for all three types of FFTP(ty-1 FFTP, ty-2 FFTP and ty-3 FFTP) as compared to the transportation cost calculated by Kumar et al. (2019) for three types of PFTP. Also, it can be observed from chart 1, chart 2 and chart 3 that our proposed methodology gives better results in Fermatean fuzzy environment for three different types of FFTP as compared with method proposed by Kumar et al. (2019) in Pythagorean fuzzy environment for three different types of PFTP. Hence, we believe that our methodology is a novel technique for dealing with imprecision in a Fermatean fuzzy setting.

6 Conclusion

One of the first and most profitable instances of LPP is TP. The TP is crucial in lot of real-life judgement scenarios. In real world, all the parameters related to TP cannot be measured precisely. Since most of the real-world situations deals with uncertainty, thus the TP in Fermatean fuzzy settings is more empirical.

First, we introduced a ranking function for ordering FFS in this research article. In order to tackle the Fermatean Fuzzy Transportation Problem (FFTP) of three different types, we also proposed an algorithm to find optimal solution. The proposed methodology has been discussed using numerical illustrations and obtained results are compared with current work. Also, by the reference graphs, we may establish that our suggested algorithm produces better outcomes than existing methods. To deal with uncertainty, the presented methodology can be applied to real-world transportation challenges.

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